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Dependence of Poisson's ratio on porosity in alumina ceramics

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Abstract The method recently developed by Asmani *et al* [*J. Euro Ceram Soc* **21** 1081 (2001)] for studying the influence of porosity on Poisson's ratio in alumina ceramics, has been revised in the present study using the Phani's model and the correct expression for the Poisson's ratio. The results obtained using the revised method are found to be in much better agreement with the experimental values of Poisson's ratio for the entire range of porosity values in alumina ceramics as compared to those obtained by Asmani *et al*

Keywords Poisson's ratio, porosity, ultrasonic velocities, alumina ceramics

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Ceramics are very useful materials for thermo-mechanical applications due to their temperature resistance, hardness, high porosity, high wear and corrosion [1-5]. The porosity of these materials affects significantly their elastic moduli such as Young's modulus, shear modulus and bulk modulus. The Poisson's ratio (σ) provides an important and useful link between different elastic moduli [6,7].

In previous studies [8-10], it has been assumed that σ remains constant for a material with different porosities. If the porosity level does not influence the Poisson's ratio, the variations of longitudinal wave velocity (V_L) and transverse wave velocity (V_T) with the change in porosity would appear to be similar. However, this is not observed experimentally in case of sintered alumina. Asmani *et al* [11] have recently measured V_l and V_T *versus* porosity using reflection method based on pulse-echo techniques. The experimental data reveal that V_L and V_T both decrease with increasing values of porosity showing a slope twice as big for the longitudinal wave [Figure 1 of Ref. 11]. In fact, ultrasonic velocity through solids depends on intermolecular and intramolecular interaction potential which depends on porosity in case of ceramic materials. The dependence of V_L on this potential is different than that for V_T. Asmani *et al* [11] have also found that Boccaccini model [9] is inadequate as it considers the similar dependences of V_l and V_t on porosity. On the other hand, the Phani's model [12,13] as further developed by Asmani *et al*, is more appropriate as it considers power dependence of V_L and V_l on porosity with different values of exponents. However, the expression for the Poisson's ratio used by Asmani *et al* suffers from an inadvertent error. We show in the present study that this error is responsible for the discrepancies between the calculated and experimental results. We obtain new formulation for the porosity dependence of the Poisson's ratio using the Phani's model.

The Poisson's ratio σ is determined from the measured values of V_l and V_T using the following relation :

$$\sigma = \frac{2V_T^2 - V_L^2}{2\left(V_T^2 - V_l^2\right)} \tag{1}$$

The expression for σ used by Asmani *et al* is different from that given by eq. (1). In the numerator, they have used $\left(2V_T^2 - 3V_l^2\right)$ in place of $\left(2V_T^2 - V_L^2\right)$. According to Phani's model [12,14], the ultrasonic wave velocity V depends on porosity P as follows

$$V = V_0(1-P)^{A} \tag{2}$$

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where V_0 is the velocity in the pore-free material ($P = 0$) and M is a constant depending on the material. By considering different behaviour for longitudinal and transverse waves, Asmani *et al* have proposed the following relation :

$$V_L = V_{L0}(1-P)^m \tag{3}$$

and

$$V_T = V_{T0}(1-P)^n \tag{4}$$

with $m \neq n$, and V_{L0}, V_{T0} are wave velocities for the pore-free material. Now, expanding the expressions for V_L^2 and V_T^2 obtained from eqs. (3) and (4) and retaining upto the quadratic terms in porosity P, we get

$$V_L^2 = V_{L0}^2\left[1 - 2mp + m(2m-1)P^2\right] \tag{5}$$

and

$$V_T^2 = V_{T0}^2\left[1 - 2np + n(2n-1)P^2\right]. \tag{6}$$

Substituting the values of V_L^2 and V_T^2 from eqs. (5) and (6) in (1), we obtain the following expression for the Poisson's ratio :

$$\sigma = \sigma_0\left[1 + AP + BP^2\right]\left[1 + aP + bP^2\right]^{-1} \tag{7}$$

with

$$A = \frac{2mV_{T0}^2 - 4nV_{T0}^2}{2V_{T0}^2 - V_{L0}^2}, \tag{8}$$

$$B = \frac{2n(2n-1)V_{T0}^2 - m(2m-1)V_{L0}^2}{2V_{T0}^2 - V_{L0}^2}, \tag{9}$$

$$a = \frac{2mV_{L0}^2 - 2nV_{T0}^2}{V_{T0}^2 - V_{L0}^2}, \tag{10}$$

$$b = \frac{n(2n-1)V_{T0}^2 - m(2m-1)V_{L0}^2}{V_{T0}^2 - V_{T0}^2} \tag{11}$$

and

$$\sigma_0 = \frac{2V_{T0}^2 - V_{L0}^2}{2\left(V_{T0}^2 - V_{L0}^2\right)}. \tag{12}$$

where σ_0 is the value of Poisson's ratio of the pore-free material. Now, expanding the last term in eq. (7) and retaining the terms upto the order P^2, we get

$$\sigma = \sigma_0\left[1 + F(m,n)P + G(m,n)\,P^2\right], \tag{13}$$

where,

$$F(m,n) = (A-a) = \frac{2(n-m)\,V_{T0}^2\,V_{L0}^2}{\left(2V_{T0}^2 - V_{L0}^2\right)\left(V_{T0}^2 - V_{L0}^2\right)} \tag{14}$$

and

$$G(m,n) = \left[(B-b) - a(A-a)\right]$$

$$= \frac{\left[(1+2n-2m)\,V_{T0}^2 - (1-2n+2m)\,V_{L0}^2\right]}{2\left(V_{T0}^2 - V_{L0}^2\right)}F(m,n) \tag{15}$$

We make use of eqs. (13-15) for estimating the values of σ in sintered alumina at different values of porosity P. The input data used are same as those given by Asmani *et al* [11]. Thus we take V_{L0} = 10904 m/s, V_{T0} = 6399m/s, m = 1.17 and n = 1.03

The value of $\sigma = \sigma_0$ at $P = 0$ turns out to be 0.2373 from eq (12). The results thus obtained in the present study using eq (13) are shown in Figure 1 alongwith the experimental values and also with the results calculated by Asmani *et al*. The results obtained in the present study using eqs. (13-15), agree almost completely with the experimental values for the entire range of porosity levels. On the other hand, the results obtained by Asmani *et al* deviate significantly from the experimental values particularly, at higher values of porosity (porosity values greater than twelve percent).

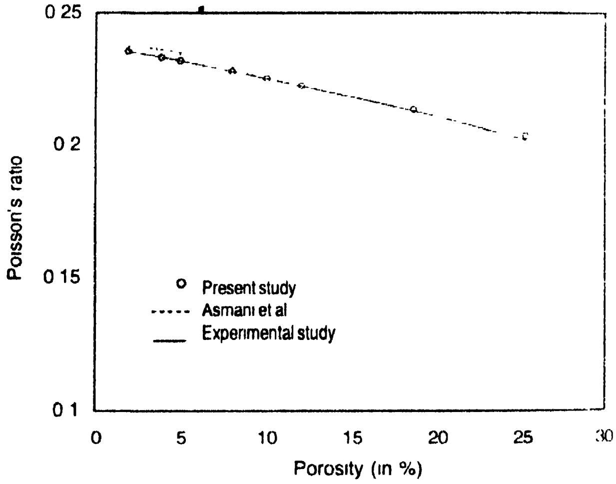

Figure 1. Dependence of Poisson's ratio on porosity

It should be mentioned that the expressions for $F(m, n)$ and $G(m, n)$ obtained in the present study eqs. (14 and 15) are different from the corresponding expressions reported by Asmani *et al*. In the present case, $F(m, n) = G(m,n) = 0$ for $m = n$, *i.e.* the Poisson's ratio does not depend on porosity when V_L and V_T depend on porosity in an identical manner. However, this condition is not satisfied by the formulation developed by Asmani *et al*. For $m = n$, $F(m,n) = 0$, but $G(m,n) = 4$ in their analysis. Thus, even in the case of $m = n$, the Poisson's ratio depends on the porosity through second order in porosity (P^2 term) in the model used by Asmani *et al*. In the present study, the equation for the porosity dependence of Poisson's ratio takes the following particular form in case of sintered alumina when the numerical values are used for $F(m,n)$ and $G(m,n)$ in eq. (13),

$$\sigma = \sigma_0 \left[1 - 0.473\, P - 0.372\, P^2\right]. \tag{16}$$

Eq. (16) should be compared with the corresponding expressions obtained by Asmani *et al* which is given below

$$\sigma = \sigma_0 \left[1 - 0.07\, P - 6.16\, P^2\right]. \tag{17}$$

It may be noted that eq. (16) used in the present study, is convergent even for the highest value of porosity (P=0.25) considered here. On the other hand, eq. (17) is divergent, *i.e.* the last term (P^2 term) dominates in the expression. The quadratic term in porosity taken by Asmani *et al*, is not adequate which is also evident from the results given in Figure 1. We have thus, demonstrated the adequacy of Phani's model in describing the variation of Poisson's ratio with porosity.

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